A Bridge – Vehicle Interaction Based Experimental Investigation of Damage Evolution

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Abstract: This paper presents an experimental monitoring of the evolution of a crack in a beam using beam-vehicle interaction response signals for identification of progressively increasing crack depth ratios. The beam is traversed by a two-axle model vehicle providing excitation in the time domain for the various extents of damage. The response of the beam in the time domain during the period of forced vibration is measured using strain gauges. A consistent evolution of damage has been demonstrated in terms of the maxima values of the measured responses. Corresponding distortions of wavelet coefficients of the measured strain data due to the presence of various levels of damage have been identified. The evolution of the phase space and the wavelet transformed phase spaces have been evaluated with damage evolution. The wavelet transformed phase spaces for the undamaged and the damaged cases are observed to be distinctly different at high scales. The importance of denoising of the acquired data and the importance of vehicle configuration have been illustrated. This study presents a basis for a general model free damage assessment and structural health monitoring framework. The presented study is particularly useful in the context of continuous online bridge health monitoring since the data necessary for analysis can be obtained from the operating condition of the bridge and the structure does not need be closed down.

Key Words: Wavelet Coefficient Maps, Distorted Ridges and Skeletons, Wavelet Transformed Phase Space, Damage Evolution, Bridge –Vehicle Interaction

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1 INTRODUCTION

The possibility of the use of bridge vehicle interaction data for structural health monitoring has gained considerable interest in recent times. A significant amount of literature of varying complexity and detail (both theoretical and experimental) is available on the bridge vehicle interaction process [1, 2, 3, 4, 5]. Although much of this deals with vehicle and axle identification problems [6, 7, 8, 9], there is a gap in the literature regarding how such data can be used best in structural health monitoring.

Majumdar and Manohar [10] have considered a bridge system with partially immobile bearings and have identified the loss of local stiffness by proposing a time domain damage descriptor. Lee et.al [11] have experimentally investigated the possible application of bridge-vehicle interaction data for identifying the loss of bending rigidity by continuously monitoring the operational modal parameters. Law and Zhu [12] have considered a simply supported beam with open and breathing cracks and discussed the dynamic behaviour of the bridge-vehicle interaction both from theory and experiment. The phase space was observed to be distorted as compared with an undamaged phase space in their study due to the presence of the crack. A number of studies [13, 14, 15, 16, 17] not related to the bridge-vehicle interaction problem have also observed that the geometry of phase space changes due to the presence of damage in the form of a crack. Bilello et.al [18] and Bilello and Bergman [19] have considered the passage of a moving mass over a damaged beam both theoretically and experimentally. Bu et.al [20] have presented a numerical study for bridge condition assessment from dynamic response of a passing vehicle considering different vehicle models, vehicle speed, sampling frequency, vehicle and bridge mass and stiffness ratios, road surface roughness, measurement noise and model error. Zhu and Law [21] have provided similar numerical studies emphasizing the importance of bridge-vehicle interaction based damage detection in concrete bridges.

These studies consistently relate to the detection, identification of the location of damage and its calibration in the presence of noise, which are the key factors affecting structural health monitoring and maintenance programmes. Such assessment, monitoring and possible predictions of damage evolution have usually been based on the analyses of the spatial or temporal response of a structure and the damage is generally quantified against a pre-existing benchmark [22]. The traditional descriptors of damage, like the change in natural frequencies are often quite small, not robust in the presence of measurement noise and fail to identify the location of damage.

The advent of sophisticated laser based devices [23, 24, 25] and the recent use of comparatively less expensive and accessible digital camera based methods [26, 27, 28, 29] in conjunction with intelligent image processing techniques and wavelet based identification of the presence, location and the extent of damage using spatial data have been reported and have gained considerable interest [30]. While these experimental studies deal with the identification of the location of damage comparatively well, very few of them investigate the evolution of the extent of damage [24, 29]. A large number of studies have been devoted to the problem of an open crack in a simply supported beam [22, 31] in this respect and the use of wavelet analysis on the damaged modeshapes [23] or static deflected shapes [28] has successfully illustrated the potential of wavelet based analyses in identifying damage without a pre-existing benchmark. The main challenge for a damage extent calibration using spatial data however lies in the difficulty of obtaining reliable and appropriate measurements in the presence of noise due to the potentially devastating masking effect [32] where the damage is overridden by the aforementioned measurement noise.

As a result, damage identification techniques in the time domain are still more popular since the data obtained is of comparatively superior quality and the measurements are generally more accessible and comparatively less complicated than obtaining data from the spatial domain. The major studies on damage identification and calibration of beams using temporal data have mostly dealt with the observation of the changes in natural frequency due to the presence of damage [33, 34, 35, 36, 37, 38] , propagation of elastic waves [39, 40], tracking of frequency contours from different modes [41] and local attractor based detections using stochastic and chaotic excitation where the structure is considered as a filter and the damage is described through phase space reconstruction [42, 43, 44, 45, 46].

All these studies are based either on free vibration or on some external forced vibration, which requires the closure of the structure especially in the case of a bridge. Thus, there exists a necessity to acquire vibration data from the bridge in its operating condition and consequently propose a practical structural health monitoring technique. It is considered that a general model – free damage detection and evolution tracking methodology has a definite potential in structural health monitoring and assessment of bridge structures under operating conditions and in the presence of measurement noise. Since small scale laboratory based experiments can be related to larger prototypes [19, 18], such experiments serve as an efficient and economic way of investigating damage evolution in bridge structures. This motivates the authors to carry out an experiment to track the evolution of damage in a model simply supported damaged beam traversed by a model two-axle vehicle. Damage is simulated in the beam by progressively increasing the extent of an open crack. Strain data at multiple locations have been acquired in the time domain during the forced vibration period. The gradual evolution has been noted first through maxima values of the measured responses at the strain gauges. Distortions of wavelet coefficients of the measured strain data due to the presence of various levels of damage have been observed. The evolution of the phase space and the wavelet transformed phase spaces have been tracked along with the evolution of damage. The potential of tracking a wavelet transformed phase space for structural health monitoring has been observed. The importance of denoising of the acquired data and the importance of vehicle configuration have been demonstrated. This study creates a framework for a general model free damage assessment and structural health monitoring which is robust against noise. Since the analysis is carried out at a signal level, the proposed approach is not limited to linear systems. The study is particularly useful in the context of continuous online bridge health monitoring since the data necessary for analysis can be obtained from the operating condition of the bridge and the structure does not therefore need be closed down.

2 BACKGROUND ON DAMAGED BEAM – VEHICLE INTERACTION

2.1 Importance of Model – Free Experimental Approach

Several models of varying complexity and detail exist for representing damage in a beam-like structure. Among these, the comparatively easier ones to detect are those which undergo a significant reduction in stiffness over a considerable region. Localised damages are often harder to detect as their global manifestation are often relatively benign. The authors have chosen a local damage to be introduced progressively to a simply supported phenolic beam in the form of an open crack. The strain and stress are considered to be maximum at the crack tip and their decay is inversely proportional to the radial distance from the tip. Such damages can be typically represented as a local reduction of the moment of inertia over a small affected width [47] or by more complex, accurate and computationally demanding representations arising from the stationarity conditions of the Hu-Washizu-Barr functional [48, 49, 50] considering the local perturbation of strain and displacement fields due to the presence of a crack. Computationally inexpensive lumped crack model of such damage is also popular among many researchers [22, 24, 51, 39, 52, 31] for simulation. All of these models contain a singularity in their modeshape, static or dynamic deflected shape, or in any of their derivatives. It is thus important to perform model free experiments at a signal level to obtain realistic changes in the time domain of the interaction between a damaged beam – like structure and a moving vehicle.

2.2 Damaged Beam Moving Load Interaction

The bridge vehicle interaction can often be modelled as a beam-moving load interaction process when the primary interest is to find an approximate nature of such interaction and the effects of the interaction on the vehicle are not important [12]. In this paper, we consider a simply supported Bernoulli Euler beam with an open crack being acted on simultaneously by n number of concentrated loads P_i (i from 1 to n) moving with an initial velocity u_0 and a constant acceleration f (Figure 1). The forced vibration equation of the problem of the damaged beam is

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \hat{c}\frac{\partial y(x,t)}{\partial t} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = \sum_{i=1}^{n} P_i \hat{\delta}(x - (u_0 t + \frac{1}{2} f t^2)) \quad (1)$$

The term $\hat{c}$ is the coefficient of damping and $\hat{\delta}$ is the Dirac-Delta function. The first fundamental mode of the beam contributes much more than the other higher modes and thus sufficiently accurate results can be obtained considering only the first mode of vibration [53] very often. Considering the first fundamental mode of the beam, by the method of separation of variables it is obtained.

$$y(x,t) = \Phi(x) q(t) \quad (2)$$

The term q(t) is the temporal response of the beam. Irrespective of the damaged model of the beam, a singularity is usually present in this modeshape in its derivative.

Multiplying both sides of equation 2 by Φ(x), integrating over the length and rearranging gives

$$\ddot{q}(t) + 2\xi\omega\dot{q}(t) + \omega^2 q(t) = \sum_{i=1}^{n} \frac{P_i}{\rho A\widehat{K}} \Phi(u_0 t + \frac{1}{2} f t^2) \qquad (3)$$

because of the sampling property of Dirac Delta function. The term ξ denotes the damping ratio of the beam and the factor $\widehat{K}$ is given as

$$\widehat{K} = \int_0^L \Phi(x).\Phi(x)dx \qquad (4)$$

It is observed, that change in the damaged modeshape gets translated into time domain on the side of forcing function during this interaction process. This is also true for a vehicle with multiple axles and multiple vehicles following each other. Introducing vehicle degrees of freedom does not change this either. When more than one number of cracks are present, the cumulative global effect of the cracks is manifested in the forcing function. Consequently, the excitation of a bridge – vehicle interaction process can be considered to be promising for a structural health monitoring process. However, it is important to note how the evolution of local damage manifests itself globally and at what level it is tangible. This reinforces the requirement of an experiment, even on a laboratory scale. The observation of global changes in the time domain is thus considered to be of significant interest.

3 EXPERIMENTS

3.1 Experimental Setup

Experiments were carried out on a 0.91m long phenolic beam with a model two - axle (axle distance 0.11m) vehicle traversing it. Figure 2 shows the general arrangement of the experiment. An open crack into the lower section of the beam was notched at a distance of 0.46m from the left hand side. The cross section of the beam was 50mm x 12mm. Two strain gauges were located at distances 0.42m (Gauge 1) and 0.52m (Gauge 2) from the left hand support of the beam respectively. The vehicle was attached to a string which in turn could be coiled around a motor. The acceleration of the vehicle could be controlled by increasing the voltage in the motor. Three damage conditions comprising crack depth ratios (CDR) of 0.167, 0.33 and 0.5 respectively were considered for the beam along with the undamaged condition. The vehicle started from rest on the beam and left the beam via an exit platform. The movement of the beam was right to left. Figure 3 shows the photograph of the arrangement. The response due to the forced vibration was recorded by the strain gauges using a commercial data acquisition software [54].

3.2 Wavelet Based Multilevel Denoising

The strain data obtained from the forced vibration of the vehicle traversing the beam was observed to contain considerable noise. Note, that there is a high frequency harmonic component (50 Hz) in the raw data which is the AC supply frequency for Ireland which has been picked up by the hardware. Additionally, there exists random additive noise to the data as well. This is a typical and a very critical problem for realistic data acquisition systems which is compounded on real structures. Generally most studies on bridge vehicle interaction and structural health monitoring problems have concentrated on additive Gaussian white noise corrupting the acquired signals and present noise stress tests [11, 10, 20, 21]. Although such studies are very useful, often non-white noise affects the signal and has therefore to be taken into account.

A wavelet based denoising in such cases can be carried out through a level dependent wavelet based estimation of the corruptive additive noise [55]. This multilevel wavelet based denoising has demonstrated superior performance in the present experiment in preference to a single level estimation of noise based on the first level coefficients on the wavelet transform. The wavelet based denoising can successfully filter out the high frequency noise as well. Figures 4a and 4b provide representative examples of the two gauges illustrating the efficiency of such denoising technique. The vehicle load and the acceleration were 7.5N and 1.1478 m/s^2 respectively. The strain data were denoised at level 6 using Coif4 wavelet basis function employing minimax algorithm and soft thresholding using the MATLAB wavelet toolbox [55].

3.3 Calibration of Damage through Strain Maxima

Law and Zhu [12] had indicated that the normalized deflections can be a sensitive indicator of the damage conditions. Since the dynamic strain can be mechanically related to the dynamic displacements, the gradual increase of the maxima values of the strain responses at the gauges has been tracked in this experiment to calibrate the correspondingly increasing damage extent. Figures 5a and 5b show the evolution of the strain responses at the two gauges for various damage conditions, while Figures 6a and 6b provide the raw and the normalized calibration values of damage respectively. The calibrations are found to be consistent. It is observed that such calibration is particularly robust against small variations of residence times of the vehicle on the beam (Figures 5 and 6). It is also noted that at these levels of damage, the global effect is tangible in some way and the stiffness of the beam reduces to a certain extent on a global basis.

3.4 Distortion of Wavelet Coefficient Maps

Melhem and Kim [56] and Kim and Melhem [57,58] reported experimental results on a concrete beam under fatigue loading and considered the wavelet transform of the acceleration response of the beam at various damage conditions against an impulse. The wavelet transform coefficient maps were observed to be significantly distorted in the presence of large damages. Figures 7a and 7b in this paper presents the absolute values of the wavelet transform coefficients of the strain data from the beam-vehicle interaction process across a range of scales at the two gauges respectively for the various extents of damage in the beam. Significant distortions in the wavelet transform are observed with the increase of damage over a range of scales. A consistent descriptor of such distortion, though non-trivial, can be helpful in characterizing the evolution of the extent damage in a structure in the time domain using this approach for damaged bridge – vehicle interaction.

3.5 Damage Evolution Tracking

Law and Zhu [12] have shown before that the phase space of the bridge-vehicle interaction becomes distorted in the presence of damage. As a direct consequence, phase space evolution tracking with increasing damage under controlled conditions of the entry and the exit of a preselected and calibrated vehicle can be helpful in terms of characterizing damage extents and the temporal evolution of a bridge. However, due to practical limitations of data acquisition or due to the unwanted participation of measurement noises corrupting the signal, it can often be difficult to obtain a meaningful insight directly from the evolution of the phase space. Even with sufficiently denoised measurements, a numerical differentiation to obtain velocity data accentuates the small noises still present in the signal, especially for cases when only the displacement or the strain data is available, as is often the case for instrumented structural bridges. A typical situation is presented in Figures 8a and 8b where the microstrain (ε) and its derivative ($\dot{\varepsilon}$) (the strain is directly related to the dynamic displacement) are plotted and the evolution of such plot is tracked for different damage conditions in the two gauges. It is extremely hard to separate the responses due to various damage severities from such a plot. In actual situations it can thus be difficult to track evolving damage from the distortions of phase space directly. To get around this difficulty, the authors propose to track the wavelet transformed phase for the monitoring of the evolution of damage. The wavelet transformed phase space filters the corrupting noise arising from measurement or from numerical operations, smoothes out the signals and retains the relative difference between the damaged and the undamaged cases from a global perspective. The wavelet transformed phase spaces for the undamaged and the damaged cases are observed to be distinctly different at high scales. To illustrate this, Figures 9a to 9g are presented to track the evolution of damage in terms of wavelet transformed phase space formed by the strain and its derivative for the first gauge, i.e. the gauge closest to the damage. The x and the y axes of Figure 9 have been indicated as $W^{(.)}\varepsilon$ and $W^{(.)}\dot{\varepsilon}$ respectively, where the term W stands for the wavelet transform and the number within the superscripted parentheses represents the scale at which the wavelet transform is performed. The phase space tracking is also important from the point of view that such tracking involves the contributions of all of the measured points significantly differing from the undamaged state at a number of scales. The development of a geometric measure of the distortion of the phase space can thus be of great importance when considering the damage model independent generality and the robustness of the approach for the purpose of calibration of damage extent. At this moment, the characterization of the best monotonous descriptor to quantify the change appears to be non-trivial. The authors have carried out tests and calibrations of damage for a number various classes of descriptors in this regard (e.g. root mean square distances from centroids of the convex hull of the transformed phase spaces) and have found out that although the results are encouraging, the calibration can be non-monotonous at certain wavelet scales. As a consequence, the authors refrain to make a conclusive statement regarding the best descriptor. The locations of strain gauges influence the tracking of damage evolution since the global distortion of the phase space would be affected if the measurement is taken too far away from the damage. There is no obvious cut-off for this distance since it also depends on the resolution of the strain gauge and the ambient noise.

6 CONCLUSION

An experimental monitoring of the evolution of an open crack in a beam is presented using a beam-vehicle interaction signal during the forced vibration period for progressively increasing crack depth ratios. A wavelet based multilevel denoising for acquired data corrupted as a preprocessing has been proposed and demonstrated to be effective. Damage calibration in terms of the maxima values of the measured responses is found to be consistent and robust against small variations of the residence time of the vehicle on the beam. The wavelet coefficient maps of the measured beam-vehicle interaction response have been found to be distorted due to the presence of damage. The evolution of increasing damage in terms of wavelet transformed phase spaces at higher scales has been demonstrated to be clearly represented than a non – transformed phase space where the effects of noise can significantly govern and mask the effects of damage evolution. The conclusions are model independent, not limited to linear problems or the problem of bridge-vehicle interaction alone. The study is particularly useful in the context of continuous online bridge health monitoring since the data necessary for analysis can be obtained from the operating condition of the bridge and the structure does not need be closed down.

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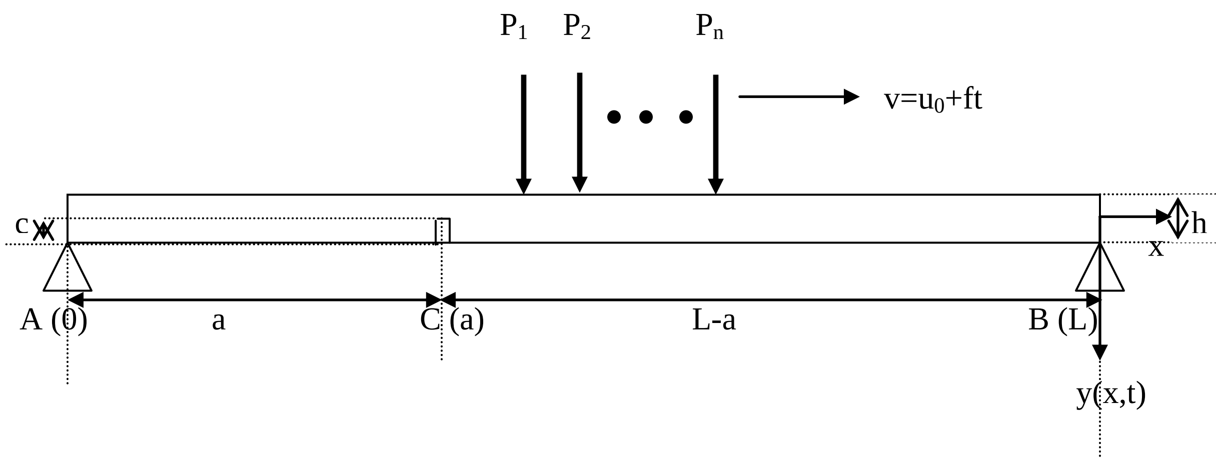


Figure 1.

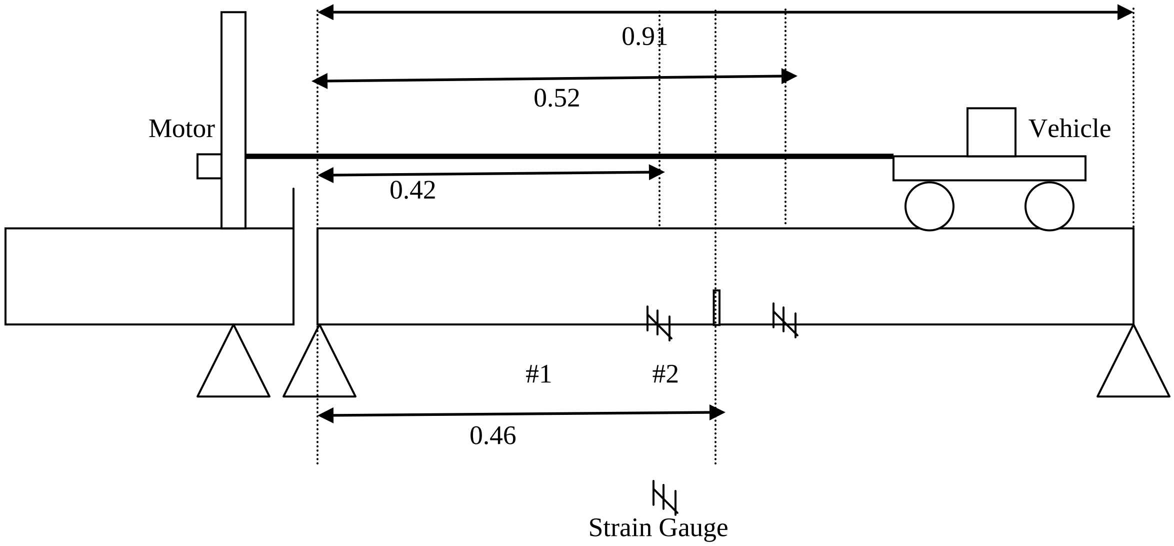


Figure 2.

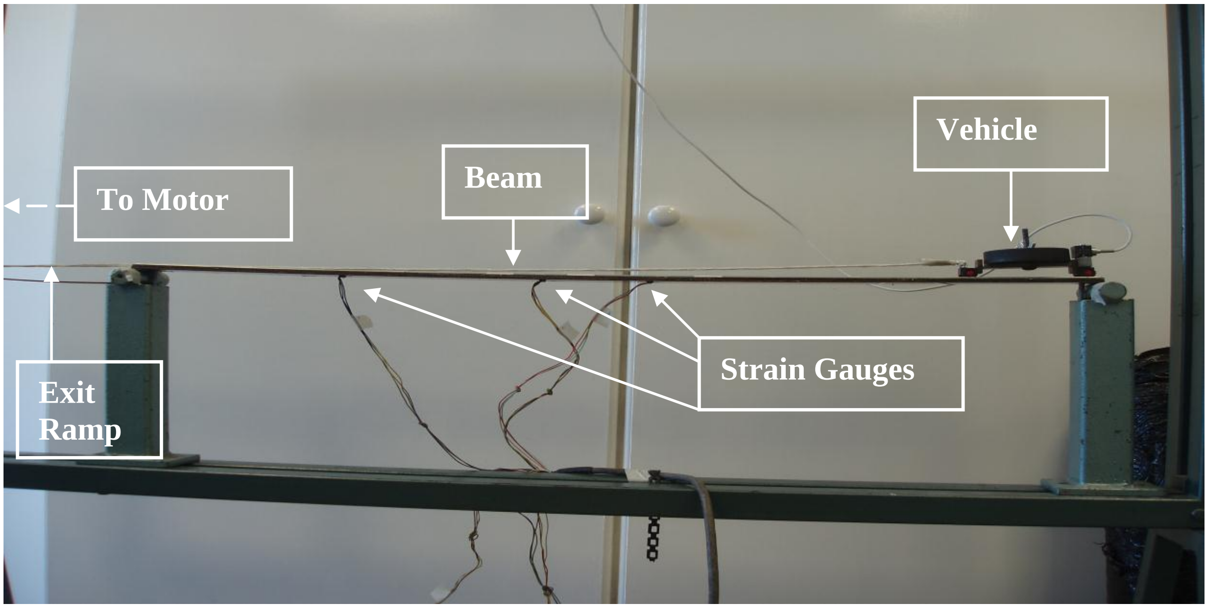


Figure 3.

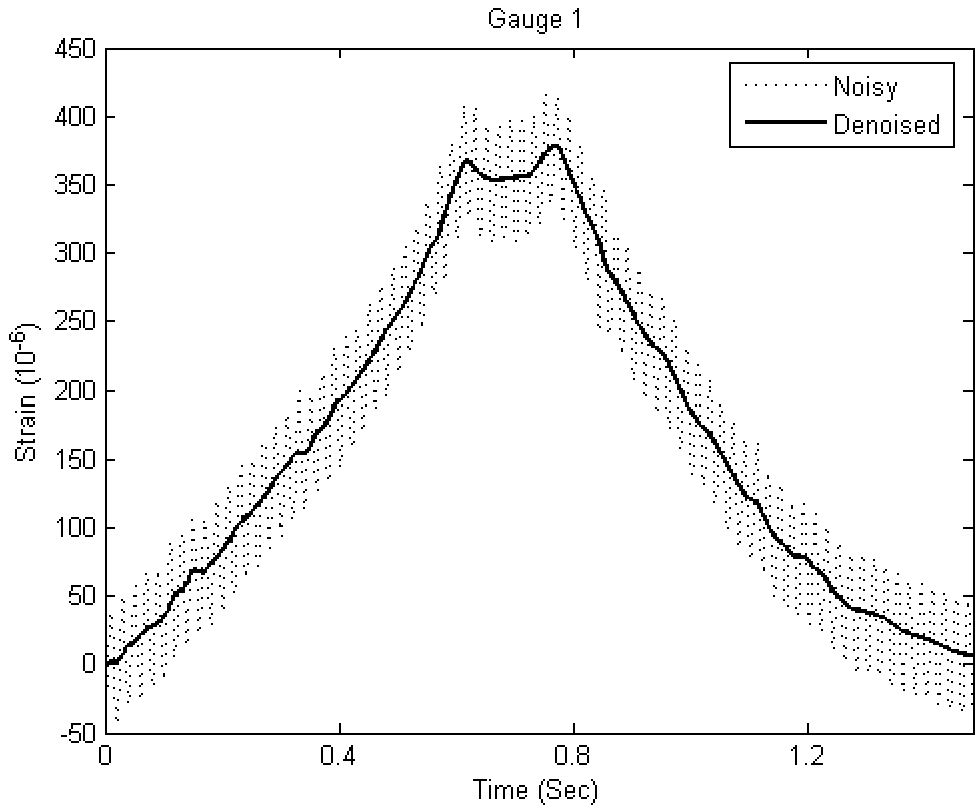

Gauge 1
Noisy
Denoised
Strain (10^-6)
Time (Sec)
450
400
350
300
250
200
150
100
50
0
-50
0
0.4
0.8
1.2

Figure 4a.

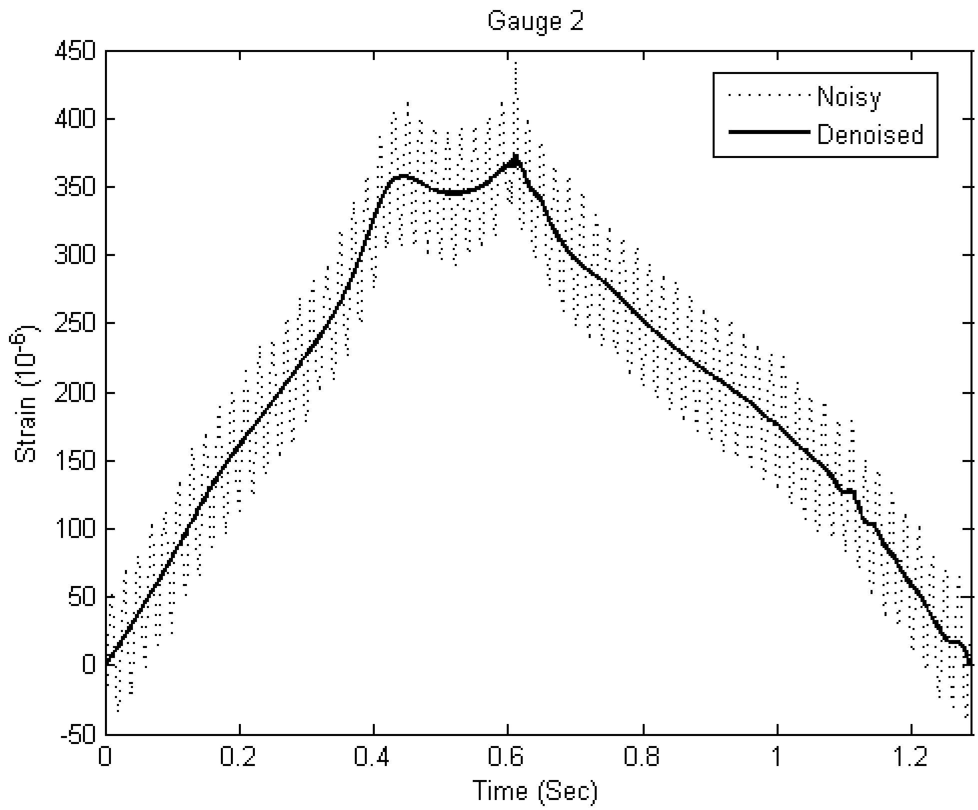

Gauge 2
Noisy
Denoised
Strain (10^{-6})
Time (Sec)
450
400
350
300
250
200
150
100
50
0
-50
0
0.2
0.4
0.6
0.8
1
1.2

Figure 4b.

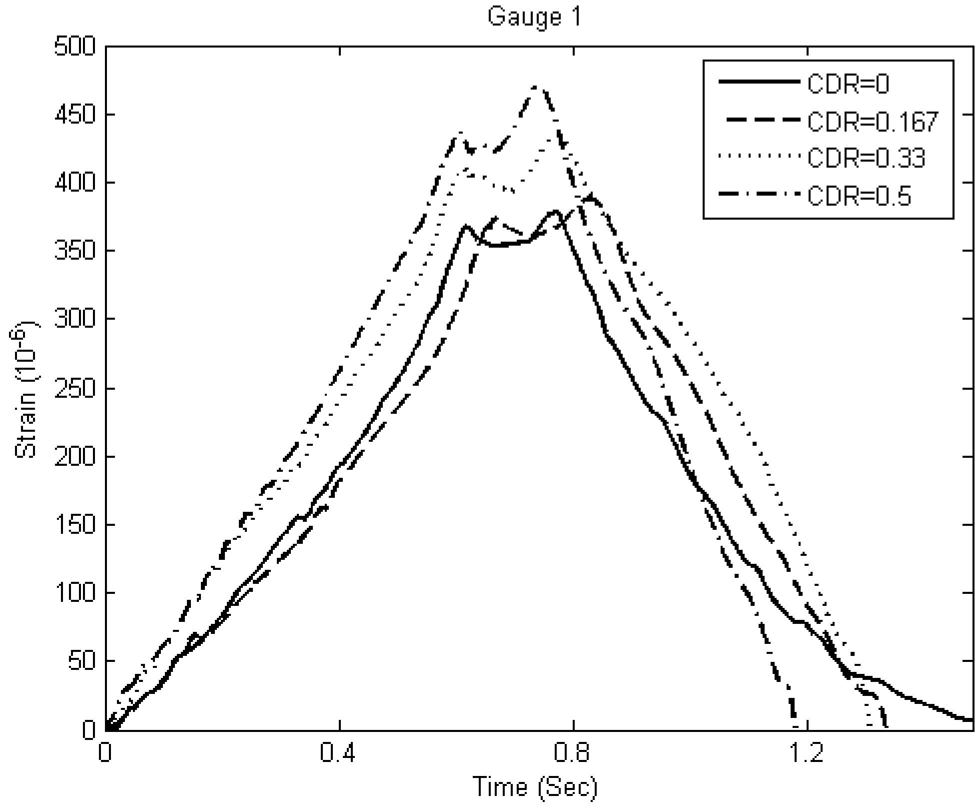

Gauge 1
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
Strain (10^{-6})
Time (Sec)
0
50
100
150
200
250
300
350
400
450
500
0
0.4
0.8
1.2

Figure 5a.

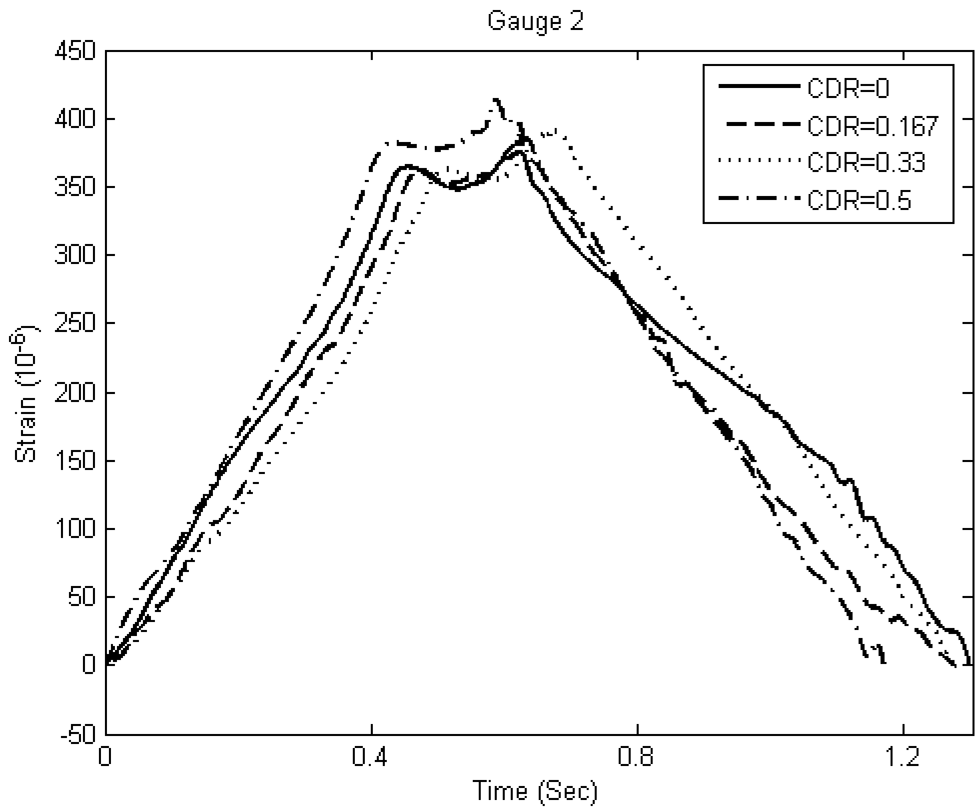

Gauge 2
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
Strain (10^-6)
Time (Sec)
450
400
350
300
250
200
150
100
50
0
-50
0
0.4
0.8
1.2

Figure 5b.

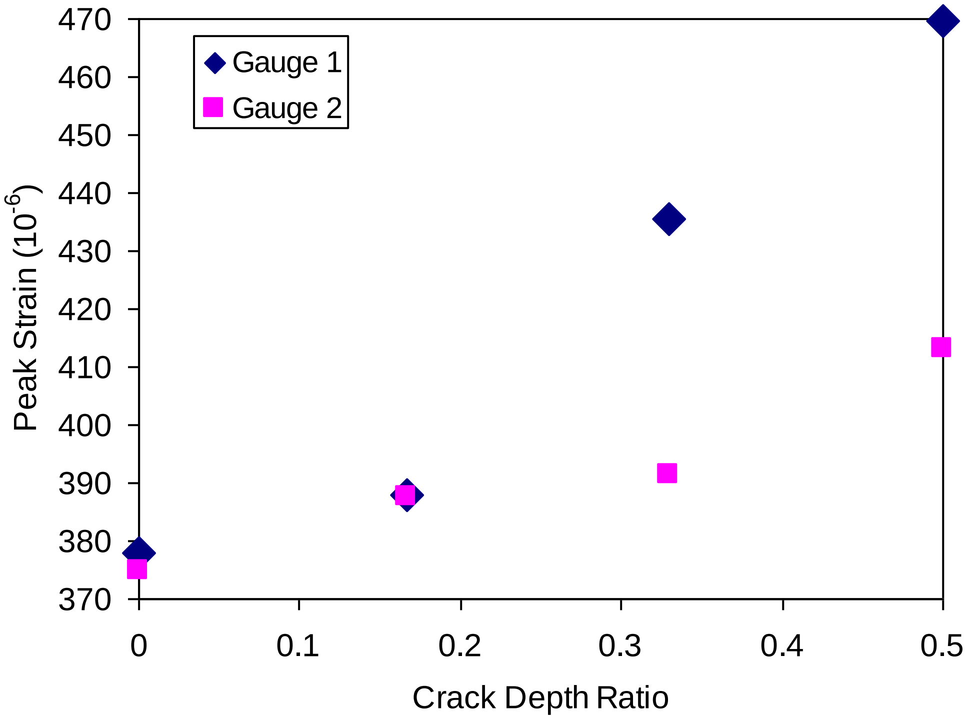


Figure 6a

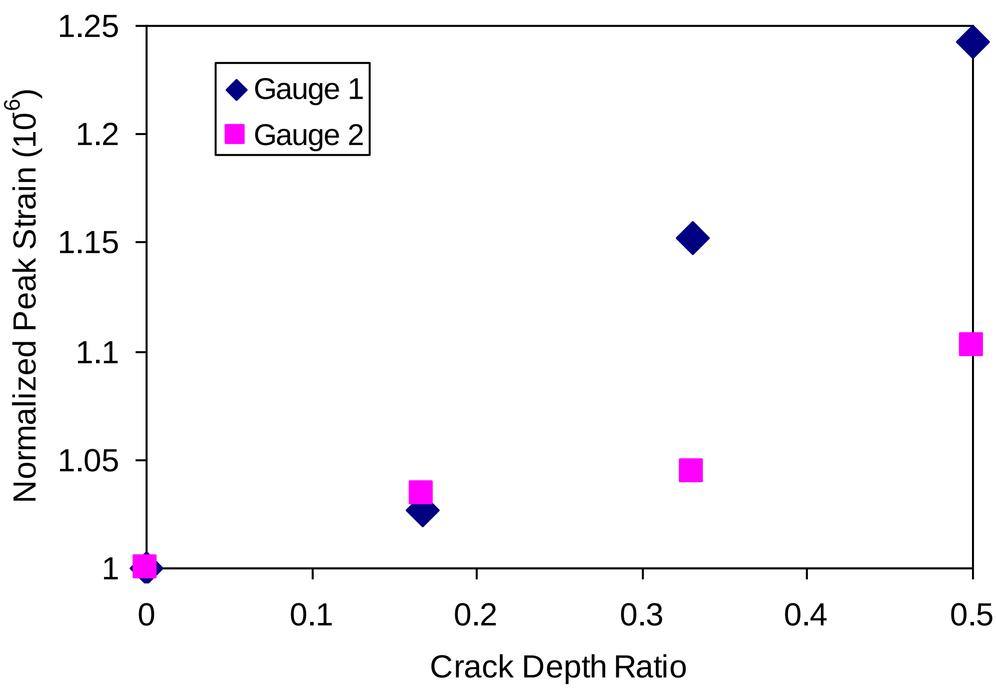
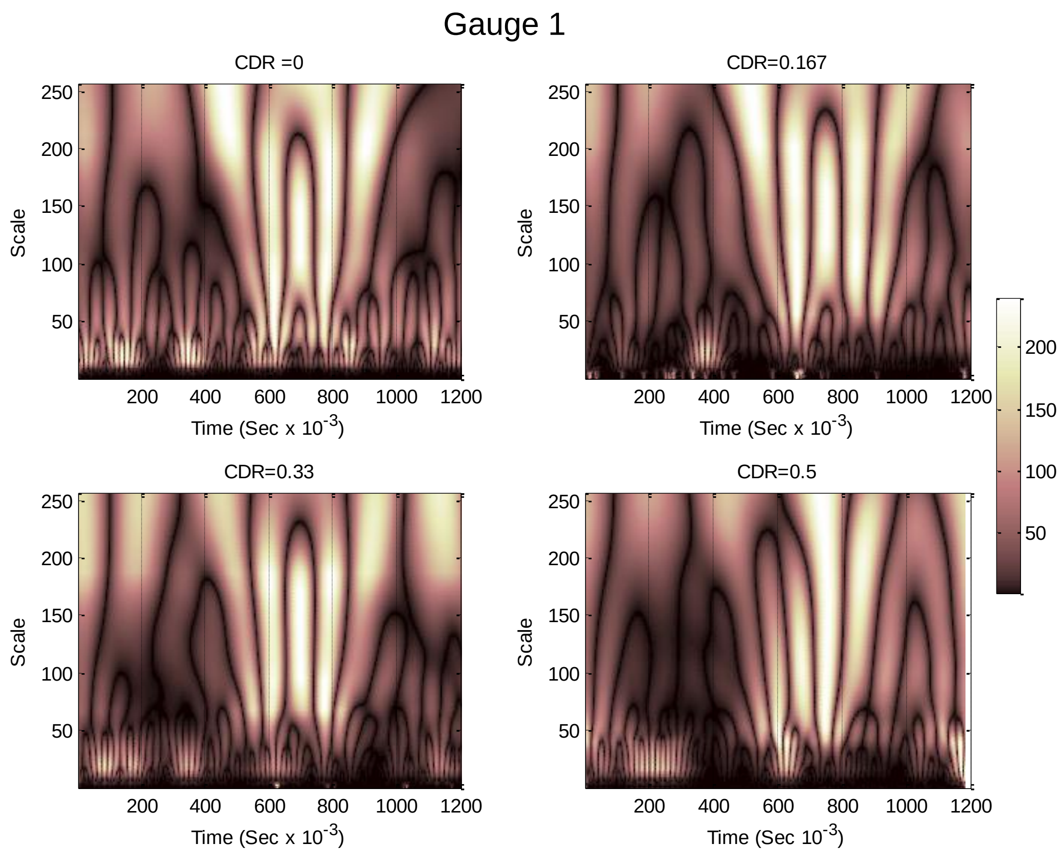


Figure 6b.



Gauge 1
CDR =0
CDR=0.167
CDR=0.33
CDR=0.5
Scale
Time (Sec x 10^-3)
Time (Sec 10^-3)

Figure 7a.

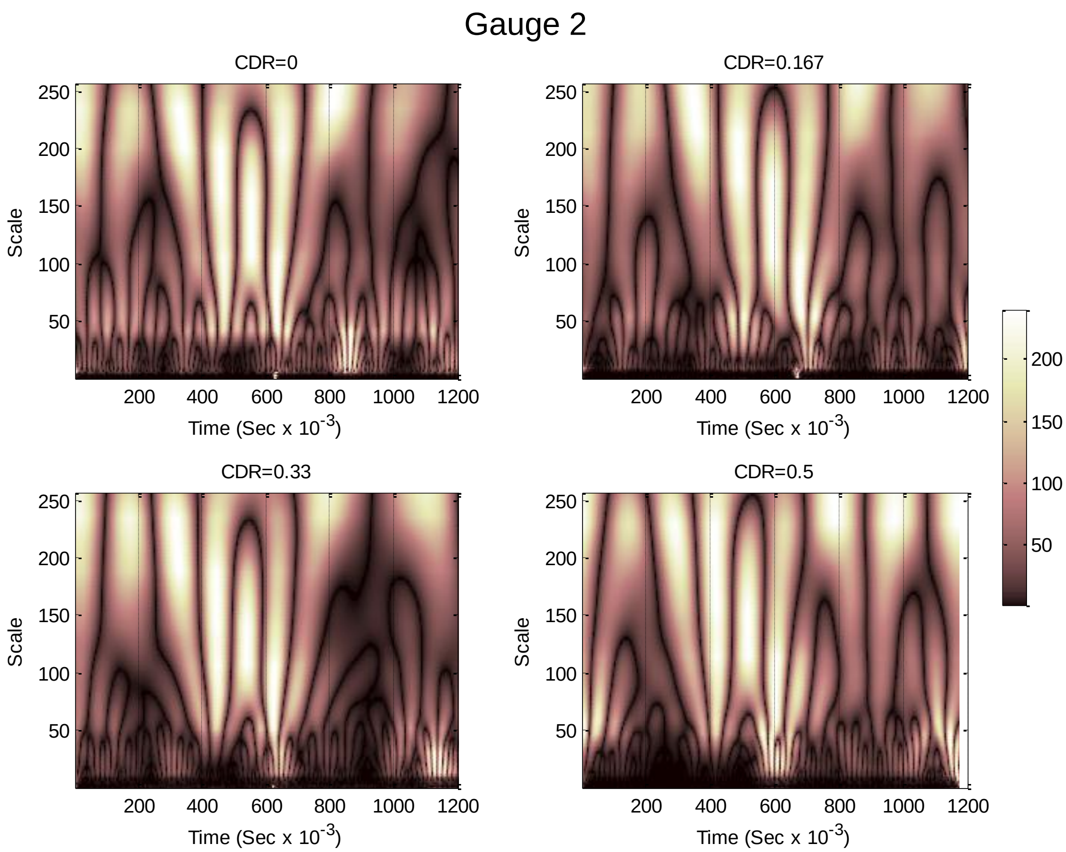

Gauge 2
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
Scale
Time (Sec x 10^{-3})
250
200
150
100
50
200
400
600
800
1000
1200

Figure 7b.

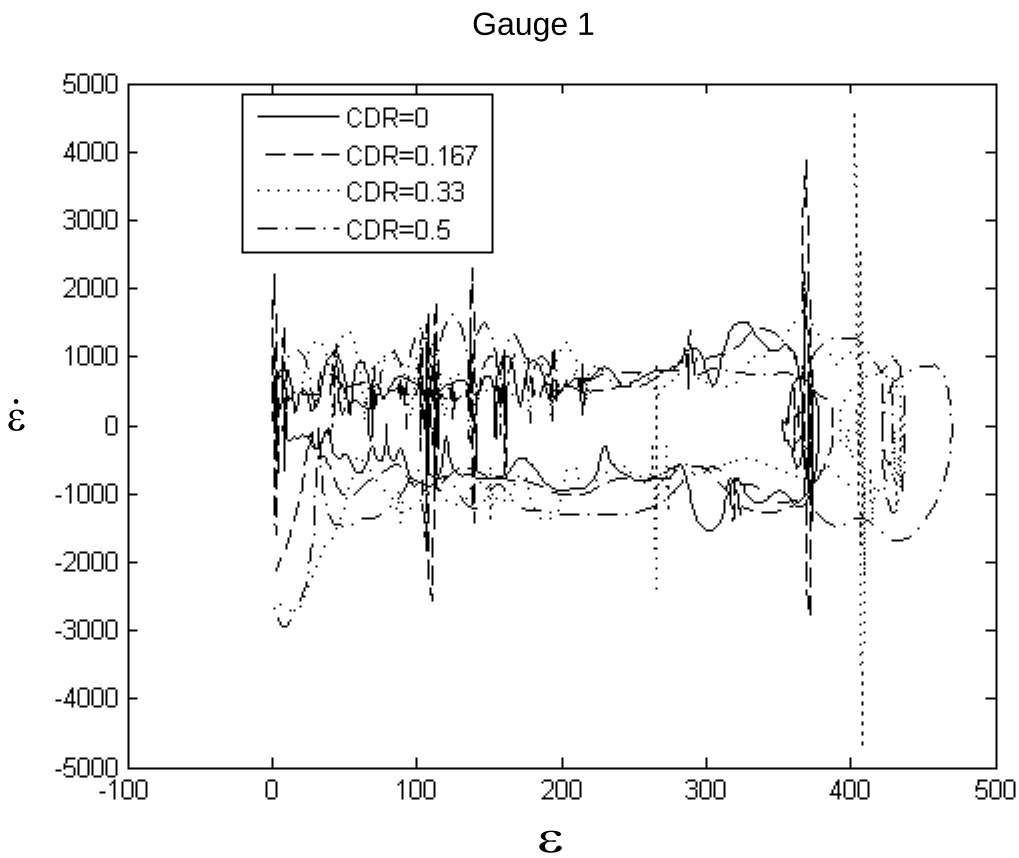

Gauge 1
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
-100
0
100
200
300
400
500
$\dot{\varepsilon}$
ε

Figure 8a.

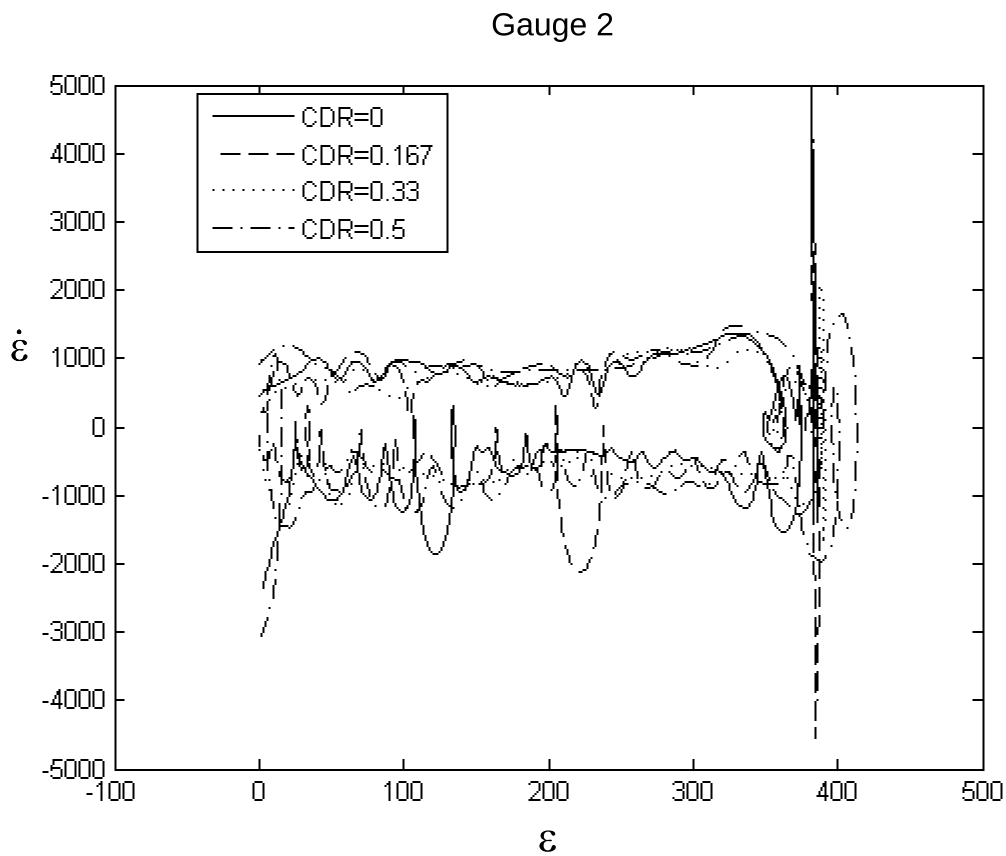

Gauge 2
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
-100
0
100
200
300
400
500
$\dot{\varepsilon}$
ε

Figure 8b.

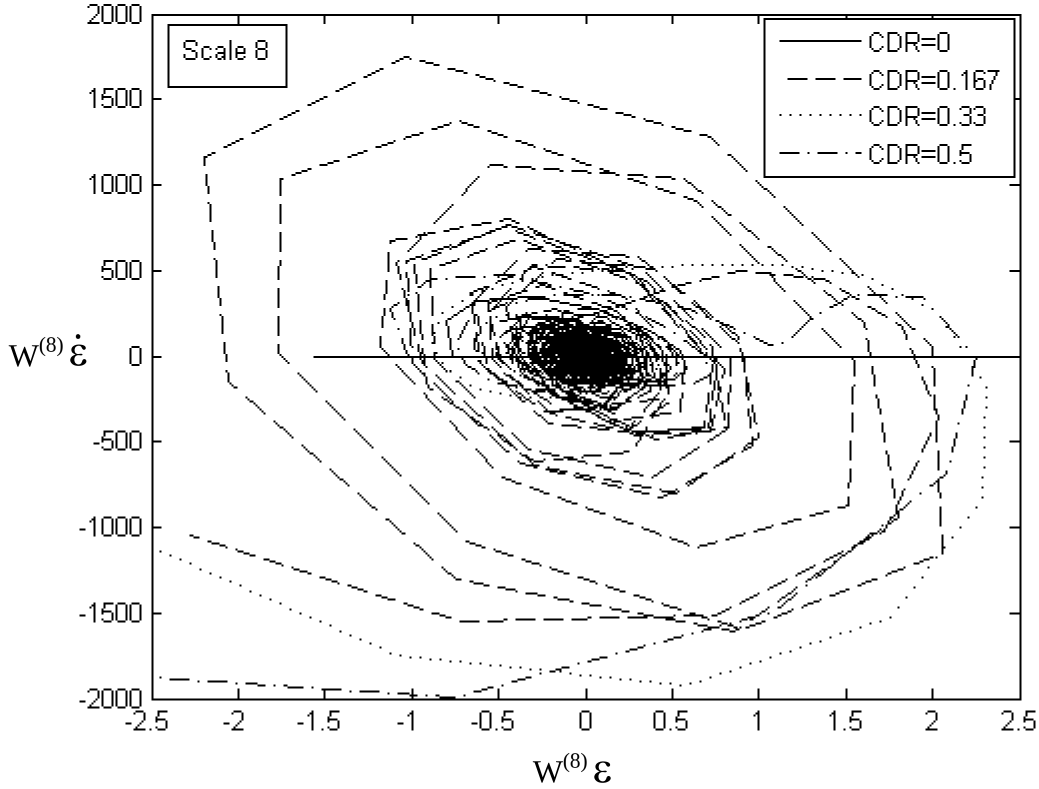

Scale 8
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
$W^{(8)}\dot{\varepsilon}$
$W^{(8)}\varepsilon$

Figure 9a.

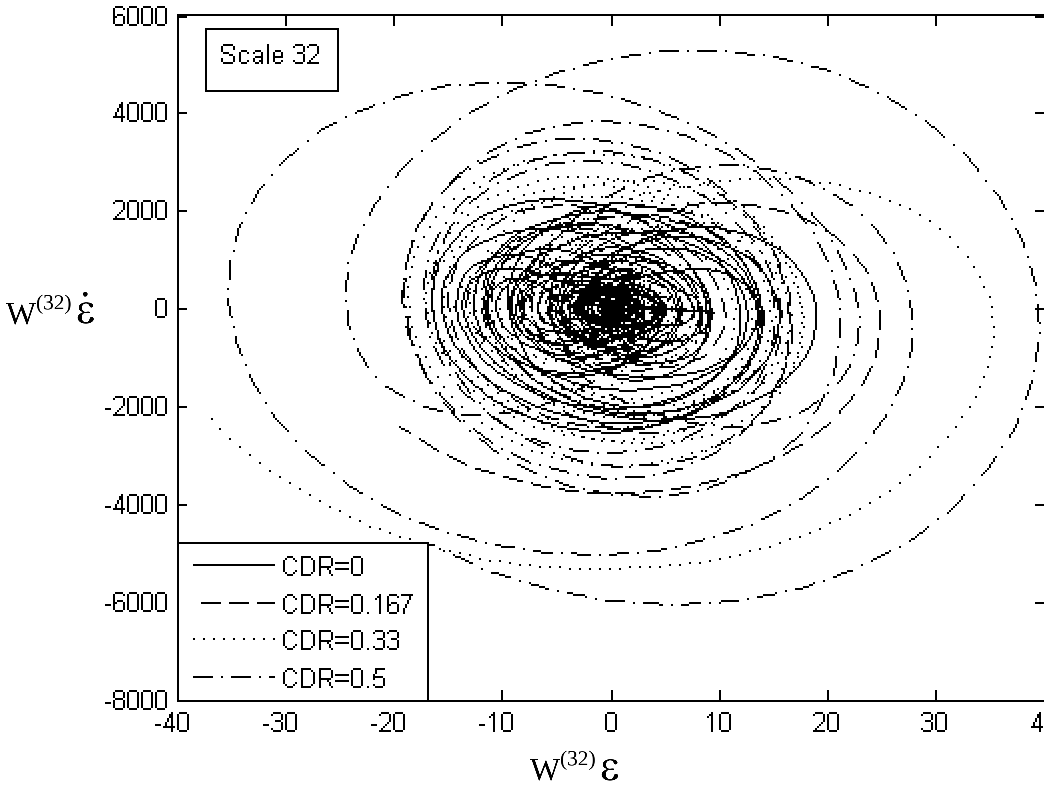
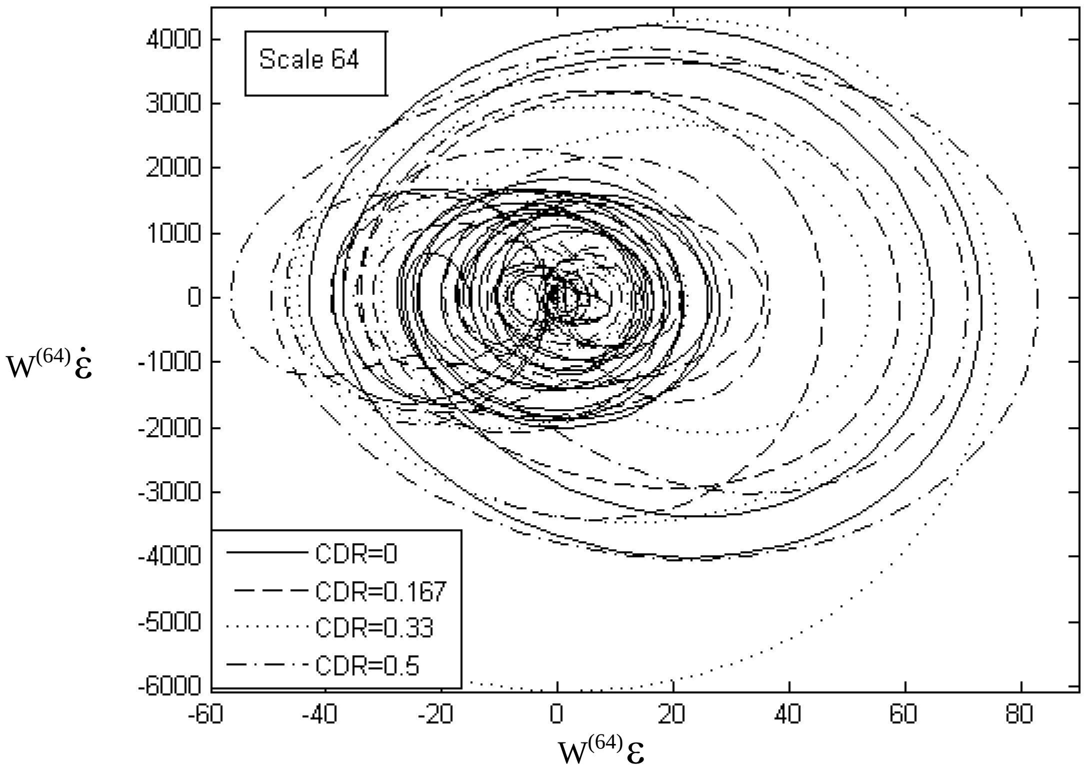

Scale 32
6000
4000
2000
0
-2000
-4000
-6000
-8000
-40
-30
-20
-10
0
10
20
30
40
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
$W^{(32)}\dot{\varepsilon}$
$W^{(32)}\varepsilon$

Figure 9b.



Scale 64
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
-6000
-60
-40
-20
0
20
40
60
80
$W^{(64)}\dot{\varepsilon}$
$W^{(64)}\varepsilon$

Figure 9c.

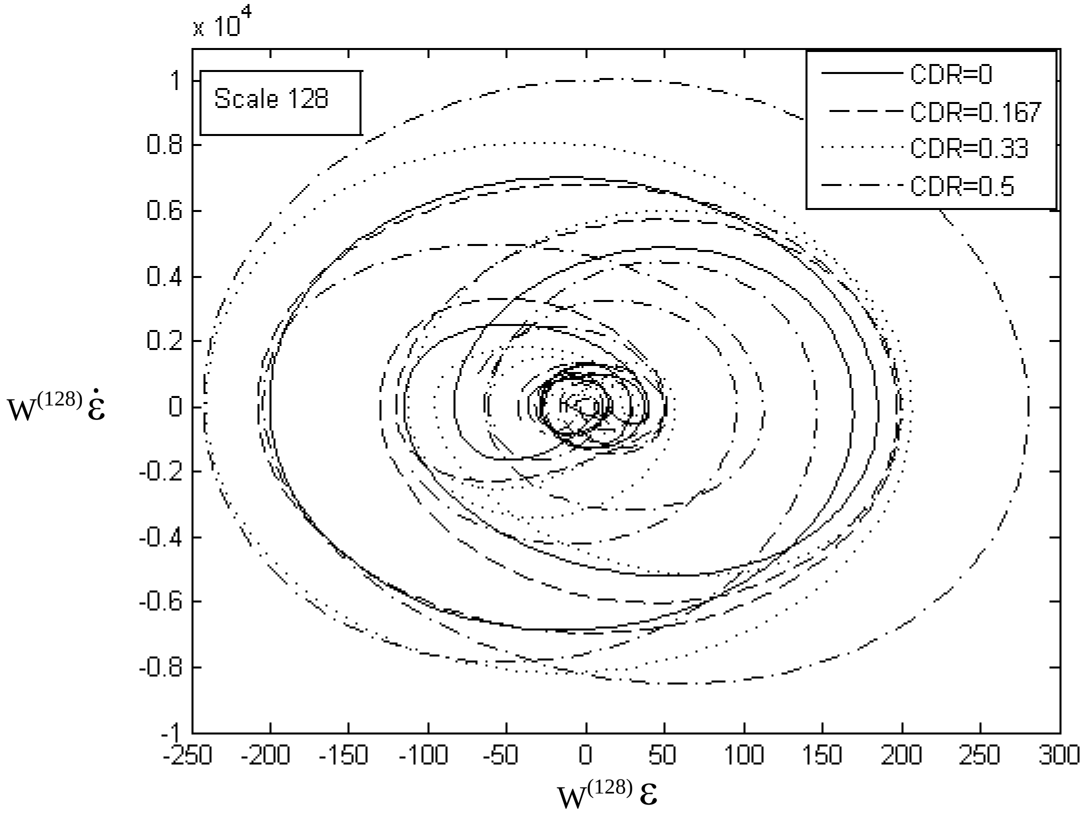
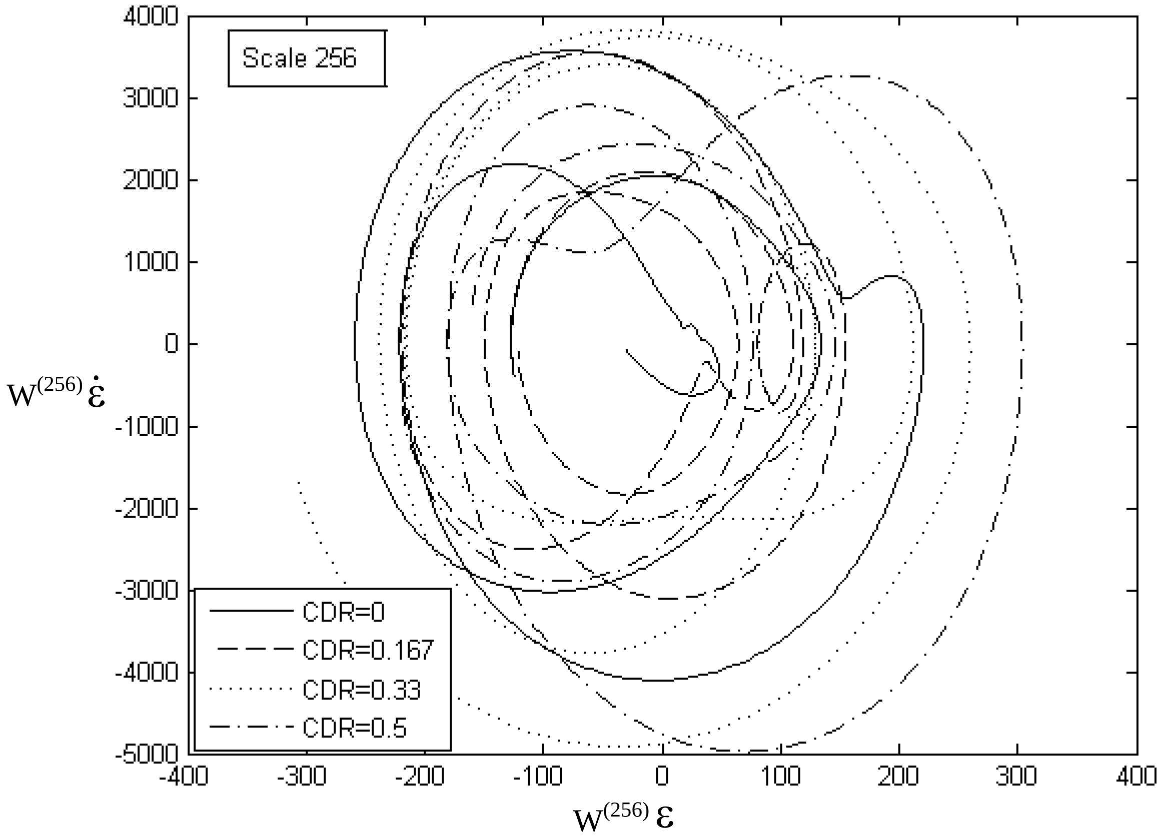

x 10^4
Scale 128
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-250
-200
-150
-100
-50
0
50
100
150
200
250
300
$W^{(128)}\dot{\varepsilon}$
$W^{(128)}\varepsilon$

Figure 9d.



Scale 256
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
-400
-300
-200
-100
0
100
200
300
400
$W^{(256)}\dot{\varepsilon}$
$W^{(256)}\varepsilon$

Figure 9e.

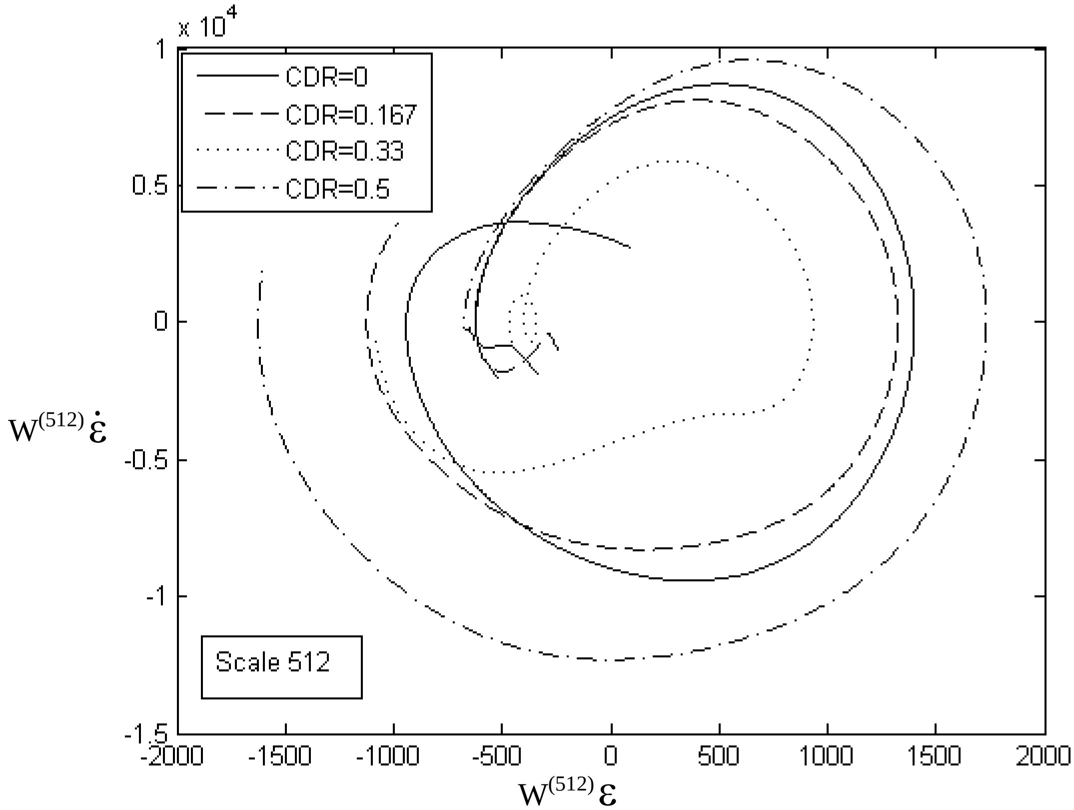

x 10^4
CDR=0
CDR=0.167
CDR=0.33
CDR=0.5
Scale 512
$W^{(512)}\dot{\varepsilon}$
$W^{(512)}\varepsilon$
1
0.5
0
-0.5
-1
-1.5
-2000
-1500
-1000
-500
0
500
1000
1500
2000

Figure 9f